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斜光格子中玻色－爱因斯坦凝聚的 Melnikov 混沌控制

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摘 要：研究了斜光格子中玻色－爱因斯坦凝聚的混沌特征，利用直接微扰法获得了 Gross-Pitaevskii 方程的微扰解。理论解析表明：这个微扰解是混沌解，因为它满足 Melnikov 混沌判据，此结果意味该系统存在混沌行为，而相应的数值模拟结果印证了其理论结果。然而，系统的混沌可以通过调节系统参数或改变初始条件加以控制，由此表明，系统参数或初始条件在控制混沌中扮演了非常重要的角色。

关键词：玻色－爱因斯坦凝聚；斜光格子；Melnikov 混沌；混沌控制

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Control the Melnikov Chaos of a Bose-Einstein Condensate in the Tilted Optical Lattices

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Abstract: The chaotic character of a Bose-Einstein condensate in titled optical lattices is investigated. A perturbation solution of the Gross-Pitaevskii equation is obtained by using the direct perturbation method. Theoretical analysis reveals that the perturbation solution is chaotic one because it satisfies the Melnikov chaos criterion, which indicates the system existing chaotic behavior. The corresponding numerical simulation results confirm the theoretical results. However, the chaos of the system can be controlled by adjusting the system parameters or changing the initial conditions, which shows the system parameters or the initial conditions play a very important role in control chaos.

Keywords: Bose-Einstein condensate; titled optical lattices; Melnikov chaos; control chaos

0 Introduction

Since a Bose-Einstein condensate (BEC) was detected in a weakly interacting gas of alkali-metal atoms held in magnetic trap[1-2], many new and important researches have been reported. These researches include the study of the stationary state properties, such as the size, shape and stabilities of the condensates[3-7]; the dynamic properties like the collective excitation and vertex[8-9]; the interesting time-dependent behaviors, the growth and collapse of the condensates[10-12], chaos[13-17], macroscopic quantum self-trapping[18-19] and the atomic tunneling between the two

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Bose-Einstein condensates (BECs) [20-22]. Recently, experiments with BECs in a Wannier-Stark(WS) system attracted much attention[23], many important theoretical studies have been reported. These works included coherent pulse output from BECs in WS system[24], the study of Bloch oscillations[25], Wannier-Stark ladders and collective tunneling effects[26]. However, these theories were based on numerical calculate method. In this paper, we give a quantitative analytical method for discussing the chaos and stability of the BECs in one-dimensional tilted optical lattice potential. We propose a perturbation solution of Gross-Pitaevskii (GP) equation by using the direct perturbation technique[15, 27], and the technique has been used successfully in some published papers [28]. Theoretical analysis results show that the perturbation solution are unbounded generally. However, if and only if a boundedness condition is satisfied, the perturbation solutions will maintain bounded. From the analysis of boundedness condition, we find the existence of a chaotic attractor related to the Melnikov chaos in the equivalent phase space, and the corresponding numerical results confirm the theoretical analysis results. Fortunately, we can control the chaos of the BEC system by adjusting the system parameters or changing the initial conditions.

1 Chaotic solutions of the BEC system

In the following we consider a trapped BEC at very lower temperature, the macroscopic wave function ψ of the condensate satisfies the self-consistent nonlinear Schrödinger equation, known as Gross-Pitaevskii equations[29-30]

$$i\hbar\frac{\partial\psi(\boldsymbol{r},t)}{\partial t}=\left[-\frac{\hbar^2}{2m}\nabla^2+V(\boldsymbol{r})+g\left|\psi(\boldsymbol{r},t)\right|^2\right]\psi(\boldsymbol{r},t), \quad (1)$$

where $V(\boldsymbol{r})$ is the trapping potential, and m the particle mass. The interaction between particles is described by a self-interaction term g=4π$\hbar^2a/m$, where a is the the atomic scattering length. For the sake of simplicity, we consider only the one-dimensional geometry $\left(\text{i.e., } \partial_y=\partial_z=0\right)$ and the trapping potential $V(\boldsymbol{r})$ is a titled optical lattice potential[13, 31]

$$V(\boldsymbol{r})=V(x)=V_0\cos(kx)+Fx, \quad (2)$$

where V_0 is the amplitude of optical potential, k the wave vector of the laser light and F the inertial force. Due to this force, a "tilted" potential is produced that leads the atoms to accelerate in x direction and make the atoms tunnel out of the traps. The Wannier-Stark Hamiltonian has the form[13]

$$H=\frac{p^2}{2m}+V_0\cos(kx)+Fx, \quad (3)$$

where m is mass of atom, x and p are the canonical position and momentum coordinates respectively. Note that in Eq.(3), for any nonzero value of F, the Stark energy Fx tends to infinity as $|x|\to\infty$, and the Hamiltonian (3) is an unbounded operator. However, for a lattice of finite size $-L\le x\le L$ with L being the finite boundary value of x or choosing the limit $F\to 0$ as $|x|\to\infty$ [32-33], the operator (3) is always bounded. In fact, the experimentally used number of 1D lattice sites are about 100 that implies $L\sim 100\pi k_i^{-1}$ [14, 34].

In order to get a simple description and a better understanding of the BEC dynamics, we employ the stationary solutions of Eq.(1) as[4, 35]

$$\psi(\boldsymbol{r},t)=\psi(x,t)=u(x)^{\mathrm{i}\theta(x)}\,\mathrm{e}^{-\mathrm{i}\mu t/\hbar}, \quad (4)$$

where μ is the chemical potential, $u(x)$ is real and normalized to the total number of particles $\int u^2(x)\,\mathrm{d}x=\int n(x)\,\mathrm{d}x=N$. Here n is the number density of the BEC atoms. Simply, we consider only the non-trivial phase and set the phase $\theta(x)$=0 in Eq.4. Substituting Eqs.(2) and (4) into Eq.(1), we can obtain the stationary state GP equation

$$-\frac{\mathrm{d}^2u(x)}{\mathrm{d}x^2}+\left[\tilde{V}_0\cos(kx)+\tilde{F}x\right]u(x)+\tilde{g}u^3(x)=\tilde{\mu}u(x), \quad (5)$$

where $\tilde{V}_0=2mV_0/\hbar^2$, $\tilde{F}_0=2mF_0/\hbar^2$, $\tilde{g}=2mg/\hbar^2$ and $\tilde{\mu}=2m\mu/\hbar^2$. It is difficult to find the exact analytical solutions of Eq.(5). However, for the weak optical potential with $V_0<<\mu$, the weak linear potential with $FL<<\mu$, we can treat the tilted lattice potential as a perturbation to seek the approximate solution of Eq.(5). We set

$$\dot{u}=\frac{\mathrm{d}u(x)}{\mathrm{d}x}=p(x), \quad (6)$$

and substitute Eq.(6) into Eq.(5), the GP equation (5) are written as

$$\dot{p}(x)=-\tilde{\mu}u(x)+\tilde{g}u^3(x)+\varepsilon(x), \quad (7)$$

Here, $\varepsilon(x)=\left[\tilde{V}_0\cos(kx)+\tilde{F}_1x\right]u(x)$, the function $\varepsilon(x)$ can be regarded as the perturbation added to the unperturbed Duffing equation

$$\dot{u}_0(x)=p_0(x),\ \dot{p}_0(x)=-\tilde{\mu}u_0(x)+\tilde{g}u_0^{\,3}(x), \quad (8)$$

Applying the Rayleigh perturbation expansion

$$u(x)=\sum_{i=0}^{\infty}u_i(x),\ |u_i|<<|u_{i-1}|, \quad (9)$$

to Eqs.(6) and (7), and equating the sum of ith-order terms to zero, we return to the zeroth-order Eq.(8) and obtain the ith-order equations

$$\begin{cases} \dot{u}_i(x) = p_i(x), \\ \dot{p}_i(x) = -\tilde{\mu} u_i(x) + 3\tilde{g} u_0^2(x) u_i(x) + \varepsilon_i(x), \\ \quad (i = 1, 2, 3, \cdots, \infty) \end{cases} \tag{10}$$

Where $\varepsilon_1(x) = \left[\tilde{V}_0 \cos(kx) + \tilde{F}x\right] u_0(x)$,

$\varepsilon_2(x) = 3\tilde{g} u_0^2 u_1(x) + \left[\tilde{V}_0 \cos(kx) + \tilde{F}x\right] u_1(x)$,

$\varepsilon_3(x) = \cdots$.

The unperturbed equation (8) possess the well-known homoclinic solution[14]

$$\begin{cases} u_0(x) = \sqrt{2\tilde{\mu}\tilde{g}^{-1}} \operatorname{sech} \xi, \\ p_0(x) = \dot{u}_0(x) = -\sqrt{2\tilde{\mu}\tilde{g}^{-1}} \operatorname{sech} \xi \tanh \xi, \end{cases} \tag{11}$$

where $\xi = \sqrt{\tilde{\mu}} x - C$, $C = \sqrt{\tilde{\mu}} x_0 - Ar\operatorname{sech} \dfrac{u_0(x_0)}{\sqrt{2\tilde{\mu}\tilde{g}^{-1}}}$, and C is the integration constant, and x_0 is the boundary point position. Combining Eq.(10) with Eq.(11) and using the variation of constant[15, 17], we easily construct formally the general solutions of Eq.(10) as

$$\begin{aligned} u_i(x) &= u'' \int_{A_i}^{x} u' \varepsilon_i(x) \mathrm{d}x - u' \int_{A_i'}^{x} u'' \varepsilon_i(x) \mathrm{d}x = \\ & u'' \int_{A_i}^{x} u' \left[\tilde{V}_0 \cos(kx) + \tilde{F}x + \tilde{g} v_i^2(x)\right] u_i(x) \mathrm{d}x - \\ & u' \int_{A_i'}^{x} u'' \left[\tilde{V}_0 \cos(kx) + \tilde{F}x + \tilde{g} v_i^2(x)\right] u_i(x) \mathrm{d}x, \end{aligned} \tag{12}$$

where $u' = \dfrac{\partial u_0(x)}{\partial x} = -\tilde{\mu}\sqrt{2\tilde{g}^{-1}} \operatorname{sech} \xi \tanh \xi$,

$$u'' = u' \int u'^{-2} \mathrm{d}x = \tilde{\mu}^{-\frac{3}{2}} \sqrt{\frac{\tilde{g}}{2}} \left(\operatorname{sech} \xi - \frac{3}{2} \xi \operatorname{sech} \xi \cdot \tanh \xi - \frac{1}{2} \sinh \xi \tanh \xi \right),$$

A_i and A_i' are the arbitrary constants depending on the initial conditions.

The derivatives of solutions u_i are given by Eq.(10) with Eq.(12) as

$$p_i(x) = \dot{u}_i(x) = \dot{u}'' \int_{A_i}^{x} u' \varepsilon_i(x) \mathrm{d}x - \dot{u}' \int_{A_i'}^{x} u'' \varepsilon_i(x) \mathrm{d}x. \tag{13}$$

Generally, the first corrections (12) are unbounded, because of the unboundedness of u'' with position x tending to infinity. Fortunately, we can easily prove that, using the l'Hospital rule, the two solutions in Eq.(12) are bounded, if and only if they satisfy the conditions[15, 22]

$$\Gamma_{i\pm} = \lim_{x \to \pm\infty} \int_{A_i}^{x} u' \varepsilon_i(x) \mathrm{d}x = 0, \; i = 1, 2, 3, \cdots. \tag{14}$$

Combining Eqs. (12) and (14) for i=1 yields the Melnikov chaos criterion

$$\begin{aligned} M(x_0) &= \Gamma_{1+} - \Gamma_{1-} = \int_{-\infty}^{\infty} u' \varepsilon_1(x) \mathrm{d}x = \\ & -\tilde{\mu}\sqrt{2\tilde{g}^{-1}} \int_{-\infty}^{\infty} \operatorname{sech}\left(\sqrt{\tilde{\mu}} x - C\right) \tanh\left(\sqrt{\tilde{\mu}} x - C\right) \cdot \\ & \left[\tilde{V}_0 \cos(kx) + \tilde{F}x\right] u_0(x) \mathrm{d}x = 0, \end{aligned} \tag{15}$$

Where x_0 is the position of the boundary point. Due to Eq. (15) shows that the Melnikov function has a simple zero, which indicates the existence of chaos and chaotic region in parameter space[15]. This means that the stable orbits given by Eq. (9) with Eqs. (11) and (13) are to be embedded in the Melnikov chaotic attractors. Therefor, we call the solutions (12) obeying the chaos criterion (15) the "chaotic solutions" [14, 28]. Applying Eqs.(11) and (12), we can obtain the particle number density of the BEC system as

$$n = u^2(x) = \left[u_0(z) + \sum_{i=1}^{\infty} u_i(x)\right]^2, \tag{16}$$

and the energy function[4]

$$\begin{aligned} E &= \int \left[\frac{\hbar^2}{2m}\left(\frac{\mathrm{d}u}{\mathrm{d}x}\right)^2 + V(x) u^2(x) + \frac{\tilde{g}}{2} u^4(x)\right] \mathrm{d}x = \\ & \int \left[\frac{\hbar^2}{2m}\left(\frac{\mathrm{d}\sqrt{n}}{\mathrm{d}x}\right)^2 + n\left(\tilde{V}_0 \cos(kx) + \tilde{F}x\right) + \frac{\tilde{g}}{2} n^2\right] \mathrm{d}x. \end{aligned} \tag{17}$$

Then the energy density of the BEC atom can be written in following form

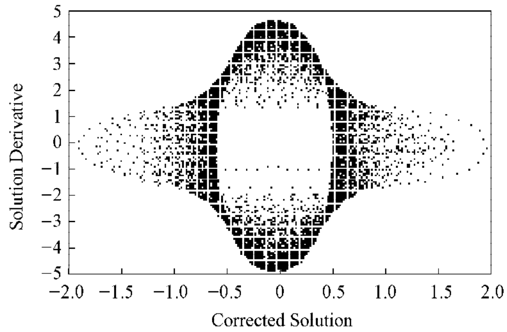
$$w = \frac{\hbar^2}{2m}\left(\frac{\mathrm{d}\sqrt{n}}{\mathrm{d}x}\right)^2 + n\left(\tilde{V}_0 \cos(kx) + \tilde{F}x\right) + \frac{\tilde{g}}{2} n^2. \tag{18}$$

From Eqs.(16) and (18), we see that the particle number density and the energy density of the system are all chaotic, because of the chaotic solutions $u_i(x)$.

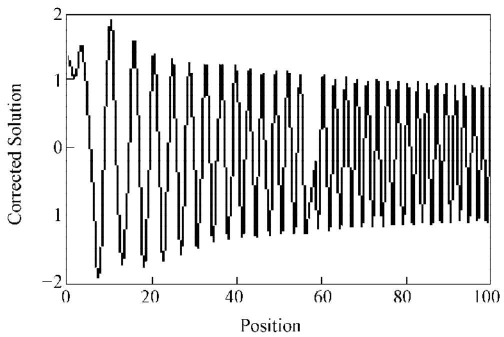
2　Control the Melnikov Chaos of the BEC System

In above section, we have obtained the chaotic solution of the BEC system by the analytical method. In order to explore this analytical insolvable system, we use numerical method to solve Eq.(5). Fix the system parameters at $\tilde{\mu} = 1$,

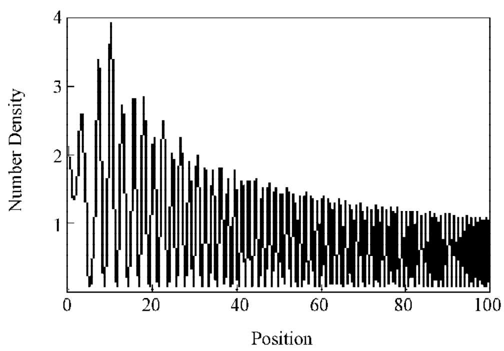
$\tilde{g}=0.5$, $\tilde{k}=1.5$, $\tilde{V}_0=0.35$ and $\tilde{F}=0.1$, we substitute them into Eq.5 and numerically plot the phase orbit in the equivalent phase space $(u, \mathrm{d}u/\mathrm{d}x)$ as shown in Fig.1 a). We see that the phase orbit exhibit the confusion from Fig.1 a), which is the chaotic feature. This indicates the chaotic behavior of the BEC system, which agrees with the theoretical analytical results. In Figs.1 b), c), and d), we plot the spatial evolution of solution, the spatial distribution of the particle number density and the energy density of the BEC system, respectively. From these figures we see that the spatial evolution and the spatial distribution of the particle number density and the energy density are very complex and confusion, which shows again the chaotic behavior of the system, in good agreement with the above theoretical analysis results again.



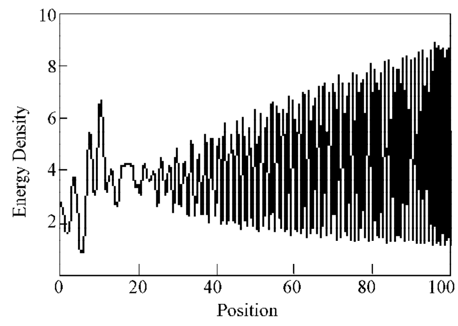
a) The chaotic phase orbits



b) The stationary solution $u(x)$



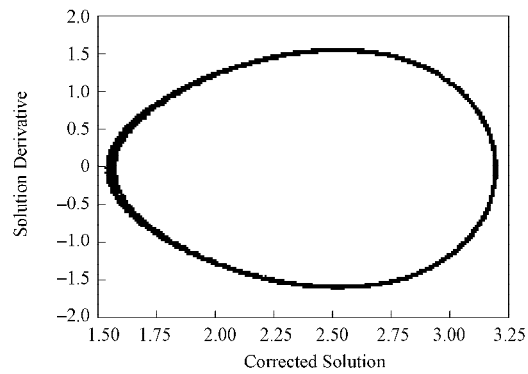
c) The particle number density n



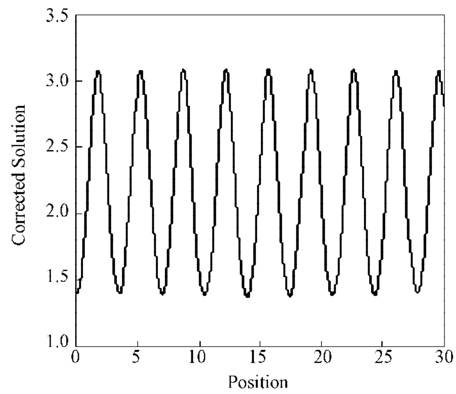
d) The energy density w

Fig. 1 Plot of the chaotic phase orbits and the chaotic spatial distribution with parameters $\tilde{\mu}=1$, $\tilde{g}=0.5$, $\tilde{k}=1.5$, $\tilde{V}_0=0.35$ and $\tilde{F}=0.1$.

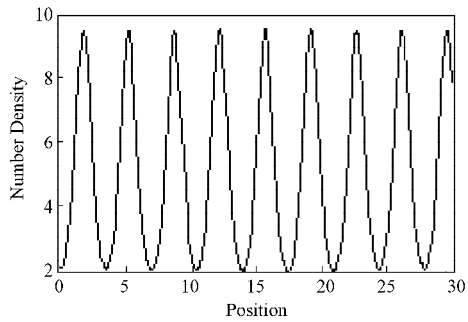
Now, we choose the controllable parameters $\tilde{\mu}=2$, $\tilde{V}_0=0.035$ and $\tilde{F}=0.0001$, with other parameters being the same as in Fig.1, and plot the chaotic phase orbits in the equivalent phase, the spatial evolution of solution, the spatial distribution of the particle number density and the energy density of the BEC system respectively as in Fig. 2.



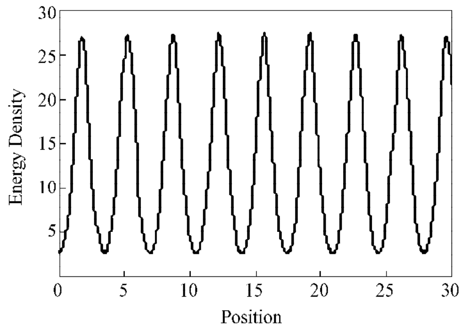
a) The periodic phase orbits



b) The stationary solution $u(x)$



c）The particle number density n



d）The energy density w

Fig. 2 Plot of the periodic phase orbits and the periodic spatial distribution with parameters $\tilde{\mu}=2$, $\tilde{g}=0.5$, $\tilde{k}=1.5$, $\tilde{V}_0=0.035$ and $\tilde{F}=0.0001$.

From Fig.1 and Fig.2 we see that phase orbits, the spatial evolution and the spatial distribution of the particle number density and the energy density are all periodic, which shows the chaos of the system have been controlled, meaning that the system parameters play a very important role in control chaos.

3 Conclusion

In this paper we have treated the chaos and the control chaos of a Bose-Einstein condensate in tilted optical lattices both analytically and numerically. We have constructed the chaotic solutions of the BEC system by using the direct perturbation method, which contain unstable periodic solutions and stable solutions embedded in the Melnikov chaotic attractors. Theoretical analysis and numerical calculation revealed that the chaos of the system depended on the system parameters or initial conditions, because of the phase orbits, the particle number density and the energy distribution of the system all displayed the sensitivity for these parameters and conditions. Therefor, we can experimentally control the chaos by adjusting the system parameters or changing the initial conditions. We believe that the technique for control chaos would be useful in other physical system.

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