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一阶格点系统的吸引子

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摘　要：通过在无穷序列空间中引入新的加权范数，证明了在耗散条件下一阶格点系统存在全局吸引子，并且得到了该全局吸引子的Kolmogorovε熵的一个上界。

关键词：格点动力系统；全局吸引子；Kolmogorovε熵

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Attractor of First Order Lattice System

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Abstract：By introducing a new weight norm in a space of infinite sequences, we proves that under some dissipative conditions the first order lattice system possesses a global attractor, and also obtains an estimate of the upper bound of the Kolmogorov's ε-entropy of the global attractor.

Key words：lattice dynamical system；global attractor；Kolmogorov's ε-entropy

1　Introduction

Lattice dynamical systems (LDSs) are spatiotemporal systems with discretization in some variables including coupled ODEs and Coupled map lattice. LDSs arise in many applications, for example, in chemical reaction theory, image processing and pattern recognition, material science, biology, electrical engineering, laser systems, etc. LDSs possess their own form, but in some cases, they arise as spatial discretizations of PDEs.

It is well known that in many cases the longtime behavior of dynamical systems, generated by evolutionary equation of mathematical physics can be described in terms of attractors of the corresponding semigroup. In LDSs, it is difficult to describe the geometric structure of the attractor and to estimate the dimension of the attractor because, generally, the attractor is infinite dimensional. One possible approach to handle this problem is to estimate the Kolmogorov's ε-entropy of the attractor[1-3].

In this paper, we consider the following lattice dynamical system

$$\begin{cases}\dot{u}_m = D(u_{m+1}-2u_m+u_{m-1})+f(u_m),\ u_m\in R^k,\ m\in Z. & (1)\\ u_m(0)=u_{m,0},\ \ m\in Z. & (2)\end{cases}$$

Where the matrix $D\in R^k$ is positive definite (and not necessarily symmetric), that is, there exists a number σ>0 such that

$$(Dv,v)\geq\sigma(v,v), \tag{3}$$

where (,) is the usual inner product in R^k, the nonlinearity f is globally Lipschitz continuous, that is, there exists $L>0$ such that

$$|f(u)-f(v)|\leq L\,|u-v|,\ \forall u,v\in R^k. \tag{4}$$

Where $|v|$ is Euclidean norm of v in R^k. And for some $\alpha,\beta>0$, $f(v)$ satisfies a dissipative estimate

$$(v,f(v))\leq-\alpha(v,v)+\beta,\ \forall v\in R^k. \tag{5}$$

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Equation(1) can be regarded as a discrete analogue of a reaction-diffusion equation in R : $u_t=Du_{xx}+f(u)$, where $u(x,t)\in R^k, D\in R^{k\times k}$ and $x\in R$.

In [1, 4], a space discretization of the equation: $u_t=\mu u_{xx}-\lambda u+h(u)+g(x)$ is studied under the assumptions $\mu,\lambda>0$ and $uh(u)\le 0$ in the standard phase space l^2. Notice that their assumption on h excludes travelling waves (in contrast to (5)) and hence allows for a compactness proof in l^2. It is clear that (4) and (5) are weaker than the assumption $uh(u)\le 0$ in dissipative sense. Obviously, $f(u)=-\alpha u+\frac{\beta u}{1+u^2}$ and $f(u)=-\alpha u+\sin u$ which satisfy (4) and (5) correspond to $h(u)=\frac{\beta u}{1+u^2}$ and $h(u)=\sin u$, respectively, which not satisfying $h(u)u\le 0$ for any $u\in R$.

In this paper, by introducing a new weight norm in space of infinite sequences, we prove that under conditions (3)~(5), the lattice dynamical system (1)~(2) possesses a global attractor, and we obtain an estimate of the upper bound of the Kolmogorov's ε-entropy of the global attractor.

2 Preliminaries

Firstly, we recall some concepts related to the absorbing set and the global attractor for a semigroup $\{S(t)\}_{t\ge 0}$ on a complete metric space H.

Definition 1[5,6] Assume that H is a complete metric space and $\{S(t)\}_{t\ge 0}$ is a semigroup of continuous operators on H. A subset $D\subset H$ is called an absorbing set for $\{S(t)\}_{t\ge 0}$ if D is bounded and every bounded set $B\subset H$ is absorbed into D in finite time. A subset Λ of a metric space H is called a global attractor for $\{S(t)\}_{t\ge 0}$ in H if Λ is a compact invariant set which attracts every bounded set in H.

In the following, we present a sufficient and necessary condition for the existence of a global attractor for the semigroup $\{S(t)\}_{t\ge 0}$ defined by general lattice dynamical systems in a Hilbert space of infinite sequences (see [7] for detail).

Let $k\in N$ be a fixed positive integer, Z denotes the set of integers. The Hilbert space of infinite sequences is $H_0=\{u=(u_i)_{i\in Z}\mid i\in Z, u_i\in R^k\}$ which endowed with the inner product $(\cdot,\cdot)_0$ and norm $\|\cdot\|_0$ as $(u,v)_0=\sum_{i\in Z}\langle u_i,v_i\rangle_0$, $\|u\|_0=(u,u)_0^{\frac{1}{2}}$, $u=(u_i)_{i\in Z}, v=(v_i)_{i\in z}\in H_0$, where $<\cdot,\cdot>_0$ is a inner product of R^k.

We consider the following lattice dynamical system with initial-value conditions in Hilbert Space H_0:

$$\begin{cases}\dot u=f(u),\ u=(u_i)_{i\in Z},\ t>0\ ;\\ u(0)=(u_{i,0})_{i\in Z}\in H_0.\end{cases}\tag{6}$$

Where $f: H_0\to H_0$ satisfy some dissipative conditions.

We assume that the solution $u(t)\in H_0$ of (6) exists globally in $R_+=[0,+\infty)$, and maps of solution $S(t): u_0\to u(t)=S(t)u(0)\in H$, $t>0$ generate a continuous semigroup $\{S(t)\}_{t\ge 0}$ on H_0, then we have the following theorem.

Theorem 1[7] The semigroup $\{S(t)\}_{t\ge 0}$ possesses a global attractor Λ in H_0 if and only if the following two conditions hold:

1) There exists a bounded set $B_0\subset H_0$ (independent of $i\in Z$) such that B_0 is an absorbing set for the semigroup $\{S(t)\}_{t\ge 0}$;

2)$\forall\varepsilon>0$, there exist $T(\varepsilon)$, $I(\varepsilon)\in N$ such that the solution $u(t)\in H_0$ of problem (6) with initial data $u_0\in B_0$, where $u(t)=(u_i(t))_{i\in Z}\in H_0$, satisfies $\sum_{|i|>I(\varepsilon)}|u_i(t)|_0^2\le\varepsilon^2$, $\forall t\ge T(\varepsilon)$,where $|\cdot|_0$ is the norm of R^k induced by the product $<\cdot,\cdot>_0$.

3 Existence and Uniqueness of Solutions

We can write (1) and (2) in the following form

$$\begin{cases}\dot u=Au+f(u), & (7)\\ u(0)=u_0, & (8)\end{cases}$$

where $u=(u_m)_{m\in Z}$, $u_m\in R^k$, the operator A is defined by $(Au)_m=D(u_{m+1}-2u_m+u_{m-1})$ and $(f(u))_m=f(u_m)$. Note that $(Au)_m=D(\partial_+\partial_-u)_m$, where

$$(\partial_+u)_m=u_{m+1}-u_m,\ (\partial_-u)_m=u_m-u_{m-1}.$$

First of all, we introduce a weight function $\rho(x)=\left(1+p^2x^2\right)^{-\gamma}$, fix $\gamma>\frac{1}{2}$, p is a parameter, which satisfies:

(I) $|\rho'(x)|,|\rho''(x)|\le c_1(p)\rho(x)$, where $c_1(p)\to 0$ as $p\to 0$.

(II) Let $\rho_m=\rho(m)=\left(1+p^2m^2\right)^{-\gamma}$, $m\in Z$, there exist constants $c_2,c_3>0$, $c_2=c_3^{-1}=e^{-\gamma}$, such that $c_2\rho_m\le\rho(m\pm 1)\le c_3\rho_m$, $\forall m\in Z$.

(III) There exists $a_1(p)>0$ such that

$J(p):=\sum\rho_m\le a_1(p)$.

In fact，by

$$\sum\rho_m=1+2\sum_{m=1}^{\infty}\left(1+p^2m^2\right)^{-\gamma}\le 1+2p^{-2\gamma}\sum_{m=1}^{\infty}m^{-2\gamma},$$

and when $\gamma>\frac{1}{2}$, $\sum_{m=1}^{\infty}m^{-2\gamma}$ is convergent, we know that $\sum\rho_m$ is convergent. Note that $\sum:=\sum_{-\infty}^{\infty}$.

(IV) It follows from (I) and (II) that there exists a function $a_2(p)$ such that $a_2(p)\to 0$ as $p\to 0$ and $|\partial_-\rho_m|\le a_2(p)\rho_m$.

(V) We fix p so small that

$$c_3 a_2(p) < \min\left\{\frac{1}{2}, \frac{\sigma}{\|D\|^2}, \alpha, 2L\right\}.$$

Below we work with this fixed p.

For $v=(v_m)_{m\in Z}$ and $w=(w_m)_{m\in Z}$, $v_m, w_m \in R^k$, let $<v,w>=\sum(v_m, w_m)$, where $(\cdot,\cdot)$ is the usual inner product of R^k. Define

$$\begin{cases} <v,w>_\rho=<v,\rho w>=\sum \rho_m (v_m, w_m), \\ \|v\|_{0,\rho}^2=<v,\rho v>=\sum \rho_m |v_m|^2. \end{cases}$$

Let Z_ρ be a space of infinite sequences $(v_m)_{m\in Z}=v$, $v_m\in R^k$, equipped with the norm $\|v\|_{1,\rho}=\|v\|_{0,\rho}+\|\partial_- v\|_{0,\rho}$ for any $v=(v_m)_{m\in Z}$.

We assume that $\tilde{f}=(f(u_m))_{m\in Z}$. By (4) and (II), we have:

$$\|\tilde{f}(u)-\tilde{f}(v)\|_{1,\rho}\le C_0\|u-v\|_{1,\rho},$$

where C_0 is a constant, i.e., $\tilde{f}(u)$ is globally Lipschitz continuous from Z_ρ to Z_ρ. By the standard theory of ordinary differential equations, we obtain the following lemma.

Lemma 1 Under the assumptions (3)~(5), for any $u_0\in Z_\rho$, there exists a unique global solution $u(t)$ of (7) ~ (8) such that $u\in C([0,+\infty), Z_\rho)$.

This establishes the existence of a dynamical system $\{S(t)\}_{t\ge 0}$ which maps Z_ρ into Z_ρ such that for each $u_0\in Z_\rho$, $S(t)u_0=u(t)$, the solution of (7)~(8).

4 Absorbing Set

Lemma 2 For any $v\in Z_\rho$, the following inequality holds:

$$<Av,\rho v>\le -\left(\sigma-\frac{a_2(p)\|D\|^2}{2}\right)\|\partial_- v\|_{0,\rho}^2+\frac{c_3 a_2(p)}{2}\|v\|_{0,\rho}^2.$$

Proof The proof is similar to that of Lemma 2.1 in [8].

Now we show that the system (7)~(8) has a bounded absorbing set in the space Z_ρ.

Lemma 3 Under the assumptions (3)~(5), there exists a bounded ball of Z_ρ, $B_{1,\rho}(0,r_0)$ centered at 0 with radius r_0, such that $B_{1,\rho}(0,r_0)$ is an absorbing set for the system (7)~(8) on Z_ρ, where $r_0=2\sqrt{\frac{\beta J(p)}{\alpha}}+2\sqrt{\frac{2L(\sigma)+2L\beta J(p)}{\alpha\sigma}}$ (independent of m). Therefore, there exists a constant $t_0\ge 0$ depending on $B_{1,\rho}(0,r_0)$ such that $S(t)B_{1,\rho}(0,r_0)\subset B_{1,\rho}(0,r_0)$, $\forall t>t_0$.

Proof Take the scalar products of both sides of the equality (7) with ρu, we have

$$<\dot{u},\rho u>=<Au,\rho u>+<\tilde{f}(u),\rho u>. \quad (9)$$

We now estimate the terms in (9) as follows.

$$<\dot{u},\rho u>=\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|u\|_{0,\rho}^2.$$

By Lemma 2,

$$<Au,\rho u>\le -\left(\sigma-\frac{a_2(p)\|D\|^2}{2}\right)\|\partial_- u\|_{0,\rho}^2+\frac{c_3 a_2(p)}{2}\|u\|_{0,\rho}^2.$$

By (5),

$$<\tilde{f}(u),\rho u>\le \beta\sum\rho_m-\alpha\sum\rho_m|u_m|^2=\beta J(p)-\alpha\|u\|_{0,\rho}^2.$$

By (V) and above inequalities, we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\|u\|_{0,\rho}^2\le -\sigma\|\partial_- u\|_{0,\rho}^2-\alpha\|u\|_{0,\rho}^2+2\beta J(p). \quad (10)$$

So, $\frac{\mathrm{d}}{\mathrm{d}t}\|u\|_{0,\rho}^2\le -\alpha\|u\|_{0,\rho}^2+2\beta J(p)$.

We assume that $\|u(0)\|_{1,\rho}\le R$ (a positive constant). Applying Gronwall Lemma to the above inequality, we have

$$\|u(t)\|_{0,\rho}^2\le R^2 e^{-\alpha t}+\frac{2\beta J(p)}{\alpha}\le\frac{4\beta J(p)}{\alpha}=r_1^2,$$

$$\forall t\ge\frac{1}{\alpha}\ln\frac{\alpha R^2}{2\beta J(p)}=t_1.$$

Take the scalar products of both sides of equality (7) with $-\rho\partial_+\partial_- u$, we have

$$<\dot{u},-\rho\partial_+\partial_- u>=<Au,-\rho\partial_+\partial_- u>+<\tilde{f},-\rho\partial_+\partial_- u>. \quad (11)$$

We now estimate the terms in (11).

$$<\dot{u},-\rho\partial_+\partial_- u>=<\rho\partial_-\dot{u},\partial_- u>+<(\partial_-\rho)T_{-1}\dot{u},\partial_- u>,$$

here and below $(T_{-1}v)_m=v_{m-1}$,

Let $L_1=<\rho\partial_-\dot{u},\partial_- u>, L_2=<(\partial_-\rho)T_{-1}\dot{u},\partial_- u>$.

Then $L_1=\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\partial_- u\|_{0,\rho}^2$.

$$<Au,-\rho\partial_+\partial_- u>\le -\sigma\|\partial_+\partial_- u\|_{0,\rho}^2.$$

$$<\tilde{f}(u),-\rho\partial_+\partial_- u>=<\rho\partial_- f(u),\partial_- u>+<\partial_-\rho(T_{-1}f(u)),\partial_- u>.$$

Let $R_1=<\rho\partial_- f(u),\partial_- u>$, $R_2=<\partial_-\rho(T_{-1}f(u)),\partial_- u>$.

Then $R_1\le L\|\partial_- u\|_{0,\rho}^2$.

Let $R'=R_2-L_2$, by $f(u_{m-1})-\dot{u}_{m-1}=-(AT_{-1}u)_m$, we have $R'=<D\partial_+\partial_-(T_{-1}u),\partial_-\rho\partial_- u>$, and similarly to Lemma 2, we can get the estimate

$$|R'|\le\frac{1}{2}(c_3 a_2(p)\|D\|^2\|\partial_+\partial_- u\|_{0,\rho}^2+a_2(p)\|\partial_- u\|_{0,\rho}^2).$$

By (V) and above inequalities, we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\|\partial_- u\|_{0,\rho}^2\le 4L\left(1+\|\partial_- u\|_{0,\rho}^2\right). \quad (12)$$

Substitute (10) into (12), we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\|\partial\_u\|_{0,\rho}^2+\frac{4L}{\sigma}\frac{\mathrm{d}}{\mathrm{d}t}\|u\|_{0,\rho}^2\le 4L+\frac{8L\beta J(p)}{\sigma}.$$

Multiplying both sides of the above inequality with $e^{\alpha t}$ and integrating on $[0,t]$, we have

$$\|\partial\_u\|_{0,\rho}^2\le\frac{(\sigma+4L)R^2}{\sigma}e^{-\alpha t}+\frac{1}{\alpha}\left(4L+\frac{8L\beta J(p)}{\sigma}\right)\le \frac{8L[\sigma+2L\beta J(p)]}{\alpha\sigma}=r_2^2,$$

$$\forall t\ge\frac{1}{\alpha}\ln\frac{\alpha(\sigma+4L)R^2}{4L[\sigma+2\beta J(p)]}=t_2.$$

Let $t_0=\max\{t_1,t_2\}$ we have that, when $t\ge t_0$,

$$\|u(t)\|_{1,\rho}=\|u(t)\|_{0,\rho}+\|\partial\_u(t)\|_{0,\rho}\le r_1+r_2=r_0.$$

The proof is completed.

5 The Global Attractor

Lemma 4 Assume that $u_0\in B_{1,\rho}(0,r_0)$ (the absorbing set of system (7)~(8)). Then for every $\varepsilon>0$, there exist $t_3(\varepsilon)$ and $I(\varepsilon)$ such that

$$\left(\sum_{|m|\ge I(\varepsilon)}\rho_m|u_m(t)|^2\right)^{\frac{1}{2}}\le\frac{\varepsilon}{2+2\sqrt{2(1+c_3)}},\ \forall t\ge t_3(\varepsilon).$$

Where $I(\varepsilon)$ and $t_3(\varepsilon)$ only depend on $\varepsilon, p, \alpha, \beta, \sigma, L$ and $\|D\|$.

Proof Choose a smooth function θ such that

$$\begin{cases}\theta(s)=0, & 0\le s\le 1;\\ 0\le\theta(s)\le 1, & 1\le s\le 2;\\ \theta(s)=1, & s\ge 2.\end{cases}\quad |\theta'(s)|\le C,\ s\in R^+,$$

where C is a positive constant. Let M be a fixed integer which will be specified later, and set $v=(v_m)_{m\in Z}$ with $v_m=\theta\left(\frac{|m|}{M}\right)u_m$.

Then taking the inner product of (7) with ρv, we get

$$<\dot{u},\rho v>=<Au,\rho v>+<f(u),\rho v>. \tag{13}$$

We now estimate the terms in (13) as follows,

$$<\dot{u},\rho v>=\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\sum\rho_m\theta\left(\frac{|m|}{M}\right)|u_m|^2.$$

$$<Au,\rho v>=-<D\partial\_u,\rho\partial\_v>-<D\partial\_u,(\partial\_\rho)(T_{-1}v)>.$$

$$<D\partial\_u,\rho\partial\_v>\ge\sigma\sum\rho_m\theta\left(\frac{|m|}{M}\right)|(\partial\_u)_m|^2+\sum\rho_m\left(D(\partial\_u)_m,\partial\_\left(\theta\left(\frac{|m|}{M}\right)u_{m-1}\right)\right).$$

Let $N=\max\{c_3,\|D\|^2\}$, we have

$$\left|\sum\rho_m\left(D(\partial\_u)_m,\partial\_\left(\theta\left(\frac{|m|}{M}\right)\right)u_{m-1}\right)\right|\le \frac{CN}{2M}\left(\|\partial\_u\|_{0,\rho}^2+\|u\|_{0,\rho}^2\right)\le\frac{CNr_0^2}{2M},\forall t\ge t_0.$$

$$|<D\partial\_u,(\partial\_\rho)(T_{-1}v)>|\le \frac{a_2(p)\|D\|^2}{2}\sum\rho_m\theta\left(\frac{|m|}{M}\right)|(\partial\_u)_m|^2+\frac{c_3a_2(p)}{2}\sum\rho_m\theta\left(\frac{|m|}{M}\right)|u_m|^2.$$

$$<f(u),\rho v>\le-\alpha\sum\rho_m\theta\left(\frac{|m|}{M}\right)|u_m|^2+\beta\sum\rho_m\theta\left(\frac{|m|}{M}\right).$$

Let $\mu=2\alpha-c_3a_2(p)$. By (V), we know that $\mu>0$. By (V) and above inequalities, we have,

$$\frac{\mathrm{d}}{\mathrm{d}t}\sum\rho_m\theta\left(\frac{|m|}{M}\right)|u_m|^2+\mu\sum\rho_m\theta\left(\frac{|m|}{M}\right)|u_m|^2\le\frac{CNr_0^2}{M}+2\beta,\ \text{for}\ \forall t\ge t_0.$$

By (III), we know that, for $\varepsilon>0$ given, there exists $I(\varepsilon)$ such that when $M\ge I(\varepsilon)$,

$$\frac{CNr_0^2}{M}+2\beta\sum_{|m|\ge M}\rho_m\le\frac{\mu\varepsilon^2}{8\left(1+\sqrt{2(1+c_3)}\right)^2}. \tag{14}$$

Therefore, we obtain that, for all $t\ge t_0$ and $M\ge I(\varepsilon)$,

$$\frac{\mathrm{d}}{\mathrm{d}t}\sum\rho_m\theta\left(\frac{|m|}{M}\right)|u_m|^2+\mu\sum\theta\left(\frac{|m|}{M}\right)\rho_m|u_m|^2\le\frac{\mu\varepsilon^2}{8\left(1+\sqrt{2(1+c_3)}\right)^2}. \tag{15}$$

By $\|u\|_{0,\rho}\le\|u\|_{1,\rho},\forall u=(u_m)_{m\in Z}$, and $\|u_0\|_{1,\rho}\le r_0$, we have $\|u_0\|_{0,\rho}\le r_0$. By (15) and Gronwall Lemma, we have, when $\|u_0\|_{1,\rho}\le r_0$,

$$\sum\rho_m\theta\left(\frac{|m|}{M}\right)|u_m(t)|^2\le e^{-\mu(t-t_0)}r_0^2+\frac{\varepsilon^2}{8\left(1+\sqrt{2(1+c_3)}\right)^2}.$$

Let $t_3=\max\left\{t_0,t_0+\frac{1}{\mu}\ln\frac{8\left(1+\sqrt{2(1+c_3)}\right)^2r_0^2}{\varepsilon^2}\right\}$, then for $t\ge t_3(\varepsilon)$ and $M\ge I(\varepsilon)$, we have

$$\sum_{|m|\ge 2M}\rho_m|u_m(t)|^2\le\sum\rho_m\theta\left(\frac{|m|}{M}\right)|u_m|^2\le\frac{\varepsilon^2}{4\left(1+\sqrt{2(1+c_3)}\right)^2},$$

which complete the proof.

Theorem 2 Under the assumptions (3)~(5), the semigroup $\{S(t)\}_{t\ge0}$ possesses a global attractor $A\subset B_{1,\rho}$ in Z_ρ, and for each $u\in A$, $u=(u_m)_{m\in Z}=(v_m)_{m\in Z}+(w_m)_{m\in Z}=v+w$,

$\| w \|_{1,\rho} \le \frac{\varepsilon}{2}$, where

$$v_m = \begin{cases} u_m, |m| \le I(\varepsilon); \\ 0, |m| > I(\varepsilon). \end{cases} \quad w_m = \begin{cases} 0, |m| \le I(\varepsilon); \\ u_m, |m| > I(\varepsilon). \end{cases} \tag{16}$$

$I(\varepsilon)$ is the least integer M satisfying the inequality (14).

Proof The existence of the global attractor $\Lambda \subset B_{1,\rho}(0, r_0)$ is easily proved by Theorem 1 and Lemma 3, 4. By Lemma 4, we know that, for each $u \in \Lambda$, $u = (u_m)_{m\in Z} = (v_m)_{m\in Z} + (w_m)_{m\in Z} = v+w$, $\| w \|_{0,\rho} \le \frac{\varepsilon}{2\left(1+\sqrt{2(1+c_3)}\right)}$, where v_m, w_m are defined by (16).

By the estimate $\| u \|_{1,\rho} \le \left(1+\sqrt{2(1+c_3)}\right) \| u \|_{0,\rho}$, we know that $\| w \|_{1,\rho} \le \frac{\varepsilon}{2}$. The proof is completed.

6 Kolmogorov's ε-Entropy of the Global Attractor

In the following, we will consider the Kolmogorov's ε-entropy of the global attractor Λ of system (7)~(8).

Definition 3[3] The Kolmogorov's ε-entropy of an attractor Λ is the logarithm of the minimal number $N_\varepsilon(\Lambda)$ of ε-balls covering the attractor in the phase space, i.e., $K_\varepsilon(\Lambda) = \ln N_\varepsilon(\Lambda)$.

Theorem 3 For any $\varepsilon > 0$,

$$Q = (2I(\varepsilon)+1)^k \ln\left(\left[\frac{2r_0\sqrt{(2I(\varepsilon)+1)^k}}{\varepsilon\sqrt{\rho I(\varepsilon)}}\right]+1\right)$$ is an upper bound of the Kolmogorov's ε-entropy of the global attractor Λ in the topology of Z_ρ, where r_0 is the same in Lemma 3, $I(\varepsilon)$ is defined by Theorem 2.

Proof The proof is similar to that of Theorem 3 in [9].

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